ON LOOSE CONNECTIVITY OF A RANDOM HYPERGRAPH

DANIEL POOLE

ABSTRACT. A loose path $P = (e_1, e_2, \ldots, e_k)$ in a hypergraph is a sequence of hyperedges such that $|e_i \cap e_{i+1}| = 1$, for all i, and $|e_i \cap e_j| = 0$ for $j \neq i, i+1$. A hypergraph is loosely connected if between any two vertices v and w, there is a loose path with v in the first edge of P and w in the last edge of P. We prove that the sharp threshold for loose connectivity in the random d-uniform hypergraph $H_d(n, m)$ is $m = \frac{n \ln n}{d}$ (along with an analogous sharp threshold for the Bernoulli random d-uniform hypergraph $H_d(n, p)$). In fact, we prove that w.h.p. the random hypergraph process, where hyperedges are added uniformly at random one after another to an initially empty hypergraph, is such that the moment that the last isolated vertex disappears is also the moment where the hypergraph process becomes loosely connected. We prove these results by analyzing a loose version of a breadth-first search on the random hypergraph $H_d(n, p)$. Along the way, we prove that the diameter of the random hypergraph process at the moment it becomes connected takes one of at most 9 explicit values around $\ln n/(\ln\ln(d-1)n)$.