Let $P_n(x) = \sum_{i=0}^n \xi_i x^i$ where the coefficients ξ_i are iid random variables. This polynomial is named after M. Kac following his seminal work on the number of real roots in the 1940s. His result (together with later refinements of other researchers) shows that if the random coefficients ξ_i are standard gaussian, then the number of real roots is, in expectation,

$$\frac{2}{\pi} \log n + C_0 + o(1)$$

where $C_0 = .65...$ is a explicit constant. The proof is restricted to the Gaussian case and no such precise estimate was known for non-gaussian polynomials. (Less precise asymptotic estimates are known due to the works of many researchers, including Littlewood-Offord, Stevens, Erdos-Offord, Ibragimov-Mashlova etc.)

Our main result gives a strong quantitative bound for the distance between consecutive real roots. In particular, we obtain an optimal bound for the probability that P_n has double real roots, when ξ_i have discrete uniform distribution (such as Rademacher).

As an application, we extend Kac's result to a large class of non-gaussian random polynomials, showing that the average of real roots is precisely $\frac{2\log n}{\pi} + C + o(1)$, where C is an absolute constant depending on the distribution of ξ_i.

Even in the Rademacher case, the value of C is not known and its determination remains a challenging question.

(joint work with Hoi Nguyen and Van Vu)