On the phase transition in random simplicial complexes

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Abstract

It is well-known that the $G(n,p)$ model of random graphs undergoes a dramatic change around $p = \frac{1}{n}$. It is here that the random graph is, almost surely, no longer a forest, and here it first acquires a *giant* (i.e., order $\Omega(n)$) connected component. Several years ago, Linial and Meshulam have introduced the $X_d(n,p)$ model, a probability space of n-vertex d-dimensional simplicial complexes, where $X_1(n,p)$ coincides with $G(n,p)$. Within this model we prove a natural d-dimensional analog of these graph theoretic phenomena. Specifically, we determine the exact threshold for the nonvanishing of the real d-th homology of complexes from $X_d(n,p)$, and show that it is strictly greater than the threshold of d-collapsibility which was determined in previous works. We also compute the real Betti numbers of $X_d(n,p)$ for $p = c/n$. Finally, we establish the emergence of giant *shadow* at this threshold. (For $d = 1$ a giant shadow and a giant component are equivalent). Unlike the case for graphs, for $d \geq 2$ the emergence of the giant shadow is a first order phase transition. The proof involves the study of the local weak limit of random simplicial complexes and its spectral measure.

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