Connectivity in bridge-addable graph classes: the McDiarmid-Steger-Welsh conjecture

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A class of graphs is *bridge-addable* if given a graph G in the class, any graph obtained by adding an edge between two connected components of G is also in the class. We prove a conjecture of McDiarmid, Steger, and Welsh, that says that if $\mathcal{G}_n$ is any class of bridge-addable graphs on n vertices, and G_n is taken uniformly at random from $\mathcal{G}_n$, then G_n is connected with probability at least $e^{-\frac{1}{2}} + o(1)$, when n tends to infinity. This lower bound is asymptotically best possible since it is reached for forests.

Previous results on this problem include the lower bound $e^{-1}+o(1)$ proved by McDiarmid, Steger and Welsh, and the successive improvements to $e^{-0.7983} + o(1)$ by Ballister, Bollobás and Gerke, and to $e^{-2/3} + o(1)$ in an unpublished draft of Norin. The bound $e^{-\frac{1}{2}} + o(1)$ was already known in the special case of bridge-alterable classes, independently proved by Addario-Berry, McDiarmid, and Reed, and by Kang and Panagiotou.

Our proof uses a "local double counting" strategy that may be of independent interest, and that enables us to compare the size of two sets of combinatorial objects by solving a related multivariate optimization problem. In our case, the optimization problem deals with partition functions of trees weighted by a supermultiplicative functional.