Intersecting families in random settings

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Abstract

Enumerating families of combinatorial objects with given properties and describing the typical structure of these objects are fundamental problems in extremal combinatorics. Using generalizations of the Bollobás set-pairs inequality and known stability results, we determine the typical structure of t-intersecting families in a variety of contexts. In each case that we examine, almost all intersecting families are trivial. This talk will focus on sparse analogues of these extremal results for families of permutations and hypergraphs, showing that they hold in random settings as well.

Balogh, Bohman, and Mubayi initiated the study of intersecting hypergraphs in the sparse random setting. Among other results, they determined the size of the largest intersecting subhypergraph of the random k-uniform hypergraph $\mathcal{H}^k(n,p)$ when $k < n^{1/2-\varepsilon}$. Hàn, Gauy, and Oliveira determined the asymptotic size of the largest intersecting family for all k and almost all p. Hamm and Kahn obtained an exact result for $k < (\frac{1}{2}-\varepsilon)(n \log n)^{1/2}$ for any constant ε, determining for which p we have with high probability that every largest intersecting subhypergraph of $\mathcal{H}^k(n,p)$ is trivial. We prove, provided p is not too small, that the same conclusion holds even for k as large as $n/4$. We remark that Hamm and Kahn also studied the case $n = 2k+1$ and $p = 1-c$ for some constant $c > 0$. We also obtain results in the context of intersecting families of permutations. Provided that p is not too small, we show that with high probability the largest t-intersecting family in $(S_n)_p$ is trivial; $(S_n)_p$ denotes the p-random subset of S_n.

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