A positive temperature Phase Transition in Random Hypergraph 2-coloring

Felicia Raßmann

Goethe University Frankfurt
(joint work with Victor Bapst and Amin Coja-Oghlan)

In this talk we establish the existence and asymptotic location of the condensation phase transition in the random hypergraph 2-coloring problem. This phase transition was predicted via the cavity method, which is an analytic but non-rigorous approach physicists have developed to put forward precise conjectures on an important class of models where the geometry of interactions between individual "sites" is determined by a sparse random graph or hypergraph. In particular we consider k-uniform random hypergraphs $H_k(n,p)$ on n vertices $V = \{1, \dots, n\}$, in which each of the $\binom{n}{k}$ possible hyperedges comprising of k distinct vertices is present with probability $p \in [0,1]$ independently. For a k-uniform hypergraph $H = (V_H, E_H)$ and a map $\sigma : V_H \to \{-1, 1\}$ we let $E_H(\sigma)$ be the number of edges $e \in E_H$ such that $|\sigma(e)| = 1$, i.e., either all vertices of e are set to 1 or to -1 under σ. Thus, if we think of σ as a coloring of the vertices of H with two colors, then $E_H(\sigma)$ is the number of monochromatic edges. The Hamiltonian E_H gives rise to a Boltzmann distribution $\pi_{H,\beta}$ on the set of all maps $\sigma : V_H \to \{-1, 1\}$: we let

$$\pi_{H,\beta}[\sigma] = \frac{\exp(-\beta E_H(\sigma))}{Z_\beta(H)}, \quad \text{where } Z_\beta(H) = \sum_{\tau : V_H \to \{-1,1\}} \exp(-\beta E_H(\tau)) \tag{0.1}$$

is the partition function. We refer to β as the *inverse temperature*. Clearly, as $\beta \to \infty$ the Boltzmann distribution $\pi_{H,\beta}$ will place more and more weight on maps σ with fewer and fewer monochromatic edges. For a given hypergraph H, the key object of interest is the function $\beta \mapsto \frac{1}{n} \ln Z_\beta(H)$, because phase transitions are exactly the non-analytic points of this function.

We are going to obtain a formula that determines the location of the condensation phase transition up to an error ε_k that tends to 0 for large k. This is the first (rigorous) result that determines the condensation phase transition within such accuracy in terms of the finite parameter β ("the positive temperature case").