A Density Turán Theorem

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Abstract

Let F be a graph which contains an edge whose deletion reduces its chromatic number. For such a graph F, Simonovits proved there exists a constant $n_0 = n_0(F)$ such that every graph on $n > n_0$ vertices with more than $\frac{\chi(F)-2}{\chi(F)-1} \cdot \frac{n^2}{2}$ edges contains a copy of F. In this paper we derive a similar theorem for multipartite graphs.

For a graph H and an integer $\ell \geq v(H)$, let $d_\ell(H)$ be the minimum real number such that every ℓ-partite graph whose edge density between any two parts is greater than $d_\ell(H)$ contains a copy of H. Our main contribution is to show $d_\ell(H) = \frac{\chi(H)-2}{\chi(H)-1}$ for large enough ℓ depending on H if and only if H has a coloring with $\chi(H) - 1$ colors such that all color classes but one are independent sets, and the exceptional class induces just a matching. When H is a clique, this recovers a result of Pfender [Complete subgraphs in multipartite graphs, Combinatorica 32 (2012), 483–495].
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