

Directed paths: from Ramsey to Ruzsa and Szemerédi

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Abstract

Starting from an innocent Ramsey-theoretic question regarding directed paths in tournaments, we discover a series of rich and surprising connections that lead into the theory around a fundamental problem in Combinatorics: the Ruzsa-Szemerédi induced matching problem. Using these relationships, we prove that every coloring of the edges of the transitive n -vertex tournament using three colors contains a directed path of length at least $\sqrt{n} \cdot e^{\log^* n}$ which entirely avoids some color. We also completely resolve the analogous question for ordinary monochromatic directed paths in general tournaments, as well as natural generalizations of the Ruzsa-Szemerédi problem which we encounter through our investigation.

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