# The phase transition of graphs embeddable on a surface of positive genus

Philipp Sprüssel

Joint work with Mihyun Kang and Michael Moßhammer

The study of random planar graphs has been initiated by the pioneering work of McDiarmid, Steger, and Welsh [3], who studied several properties of a graph drawn uniformly at random from the class of all planar graphs on $n$ vertices, like connectivity, degrees, and automorphisms. They were also the first to show that the number of planar graphs on $n$ vertices has an exponential growth rate $\gamma$, the value of which was later determined by Giménez and Noy [1]. The latter also obtained limit laws for planar graphs with $n$ vertices and $m = \mu n$ edges for $\mu \in (1, 3)$.

In view of the classical Erdős-Rényi random graph, the more interesting regime is $\mu \sim \frac{1}{2}$: at this edge density, a unique component of linear size—the *giant component*—appears in a graph drawn uniformly at random from all graphs with $n$ vertices and $m = \mu n$ edges, the so-called *phase transition* of the Erdős-Rényi random graph. Kang and Łuczak [2] showed that random *planar* graphs not only feature an analogous phase transition at $\mu \sim \frac{1}{2}$, but surprisingly undergo a second phase transition at $\mu \sim 1$, when the giant component covers nearly all vertices.

We show that a random graph embeddable on an orientable surface $\mathbb{S}_g$ of positive genus $g$ (i.e. a sphere to which $g$ handles have been attached) undergoes phase transitions similar to those of planar graphs. We further show other structural properties of random graphs embeddable on $\mathbb{S}_g$: for instance, the giant component (as soon as it appears) in such a graph is not embeddable in a surface of lower genus with high probability, while all other components are planar.

## References

[1] O. Giménez and M. Noy. Asymptotic enumeration and limit laws of planar graphs. *J. Amer. Math. Soc.*, 22(2):309–329, 2009.

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