The First Order World of Galton-Watson Trees

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Preliminary Report

Let T_λ denote the usual Galton-Watson Tree with Poisson mean λ children. For any property A let $f_A(c)$ denote the probability that T_λ has the property.

The first order language consists of equality $=$, the constant root R, and $\pi(x, y)$, that x is the parent of y. We allow the usual Boolean connectives and universal and existential quantification over *vertices*.

We show that any first order A can be changed to an A' which depends only on the local neighborhood of the root so that $f_A(\lambda) = f_{A'}(\lambda)$ for all λ. We show that $f_A(\lambda)$ is the *unique* solution to a finite system of equations. We give an explicit mapping $\Psi_\lambda : D \to D$ where D is the set of distributions of a finite set Σ. Ψ_λ will be a contracting map and $f_A(\lambda)$ will be expressed in terms of its unique fixed point. Further $f_A(\lambda)$ is a real analytic function. In stark contrast: let A be that T_λ is finite. Then $f_A(\lambda)$ in not differentiable at the critical point $\lambda = 1$.