

BIRTHDAY INEQUALITIES, HARD SPHERES, AND INDEPENDENT SETS

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ABSTRACT. I will discuss a *birthday inequality* in random geometric graphs: the probability of the empty graph is upper bounded by the product of the probabilities that each edge is absent. I show the birthday inequality holds at low densities, but does not hold in general. As an application, I give a new bound on the number of independent sets of a given size in d -regular graphs.

The birthday inequality is implied by a *repulsion inequality*: the expected volume of the union of spheres of radius r around n randomly placed centers increases if we condition on the event that the centers are at pairwise distance greater than r . Surprisingly, the repulsion inequality is not true in general, and in particular that it fails in 24-dimensional Euclidean space: conditioning on the pairwise repulsion of centers of 24-dimensional spheres can *decrease* the expected volume of their union.