Bootstrap percolation process on inhomogeneous random graphs

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Bootstrap percolation introduced by Chalupa, Leath and Reich (1979) is a deterministic infection process where in every round each vertex with at least r infected neighbours becomes infected. Once a vertex is infected it remains infected forever. The process stops when no more vertices can become infected. We consider the behaviour of the process on the so-called Chung-Lu random graphs, when the set of initially infected vertices is chosen at random. The Chung-Lu random graph model is an inhomogeneous random graph where every vertex is given a weight and two vertices are connected with probability proportional to the product of their weight.

Our aim is to examine the total number of infected vertices. We say that a linear outbreak occurs if a linear fraction of the vertices become infected. Certain weight sequences exhibit a threshold behaviour for this property. In particular there exists a sublinear weight sequence such that if the initial number of infected vertices is below the threshold then with high probability (w.h.p.) no linear outbreak occurs, but when the initial number of infected vertices exceeds the threshold then w.h.p. a linear fraction of the vertices will become infected. However this threshold behaviour is not present for every weight sequence. In these cases for a linear outbreak to occur the size of the initial infection set already has to be linear. We determine conditions on the weight sequence for the threshold behaviour to exist. In addition we establish the threshold on the number of initially infected vertices for these weight sequences.

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