Diameter of a random distance graph

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We introduce a class of graphs $G(n, r, s) = (V(n, r), E(n, r, s))$ defined as follows:

$$V(n, r) = \{x = (x_1, x_2, \ldots, x_n) : \ x_i \in \{0, 1\}, \ x_1 + x_2 + \ldots + x_n = r\},$$

$$E(n, r, s) = \{\{x, y\} : (x, y) = s\},$$

where (x, y) is the Euclidean scalar product. Such graphs play an important role in combinatorial geometry, Ramsey theory, and coding theory.

We study the diameters of random graphs $G_p(n, r, s)$ whose edges are chosen independently from the set $E(n, r, s)$, each with probability p. We obtain sharp asymptotic bounds for corresponding threshold probabilities depending on some relations between the parameters n, r, s.