Tight cycles and regular slices in dense hypergraphs

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Joint work with Peter Allen, Julia Böttcher and Oliver Cooley

We prove a hypergraph analogue of the Erdős-Gallai theorem: for any $\delta > 0$ there exists n_0 such that for any $0 \leq \alpha \leq 1$, any k-uniform hypergraph H on $n \geq n_0$ vertices with at least $(\alpha + \delta)\binom{n}{k}$ edges contains a tight cycle of length at least αn. Moreover, H contains tight cycles of all lengths up to αn which are divisible by k.

A key tool in our proof is a 'regular slice lemma', which is derived from the strong hypergraph regularity lemma by selecting eges according to a random process. Given a large k-uniform hypergraph G, this returns a sparse subgraph of G which accurately represents G (for example, degree conditions and densities of small subgraphs are proportionally inherited) and which has useful regularity properties. In particular, we can give a natural definition of a 'reduced k-graph' of G which captures the structure of G. We believe that, compared to the use of the strong hypergraph regularity lemma, using our 'regular slice lemma' allows considerable simplification in existing proofs in the field of extremal hypergraph theory, and also that these arguments are closer in flavour to arguments using the regularity lemma for graphs.

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