High degrees in recursive trees

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Abstract

Let T_n be a random recursive tree with vertex set $[n] := \{1, \ldots, n\}$ and edges directed towards the root. Let $\deg_n(i)$ denote the number of children of vertex $i \in [n]$ of T_n. Devroye and Lu [1] showed that the maximum degree Δ_n of T_n satisfies $\Delta_n / \log_2 n \to 1$ a.s. Here we study the distribution of the maximum degree and of the number of vertices with near-maximum degree.

For any $d \in \mathbb{Z}$, let $X_d^{(n)} = |\{i \in [n] : \deg_n(i) = \lfloor \log_2 n \rfloor + d\}|$. Also, let $\mathcal{P}$ be a Poisson point process on $\mathbb{R}$ with rate function $\lambda(x) = 2^{-x} \cdot \ln 2$. We show that, up to lattice effects, the vectors $(X_d^{(n)}, d \in \mathbb{Z})$ converge in distribution to $(|\mathcal{P} \cap [d, d+1)|, d \in \mathbb{Z})$. This recovers and extends results of Goh and Schmutz [2].

Keywords: Random trees, Random recursive trees, Kingsman's coalescent, Union-Find, Analysis of algorithms.

References

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[2] W. Goh and E. Schmutz, *Limit distribution for the maximum degree of a random recursive tree.* In Journal of Computational and Applied Mathematics **142** (2002), pages 61–82.