Robust hamiltonicity of random directed graphs

- abstract -

In his seminal paper from 1952 Dirac showed that the complete graph on $n \geq 3$ vertices remains Hamiltonian even if we allow an adversary to remove $\lfloor n/2 \rfloor$ edges touching each vertex. In 1960 Ghouila-Houri obtained an analogue statement for digraphs by showing that every directed graph on $n \geq 3$ vertices with minimum in- and out-degree at least $n/2$ contains a directed Hamilton cycle. Both statements quantify the robustness of complete graphs (digraphs) with respect to the property of containing a Hamilton cycle.

A natural way to generalize such results to arbitrary graphs (digraphs) is using the notion of *local resilience*. The local resilience of a graph (digraph) G with respect to a property $\mathcal{P}$ is the maximum number r such that G has the property $\mathcal{P}$ even if we allow an adversary to remove an r-fraction of (in- and out-going) edges touching each vertex. The theorems of Dirac and Ghouila-Houri state that the local resilience of the complete graph and digraph with respect to Hamiltonicity is $1/2$. Recently, this statements have been generalized to random settings. Lee and Sudakov (2012) proved that the local resilience of a random graph with edge probability $p = \omega(\log n/n)$ with respect Hamiltonicity is $1/2 \pm o(1)$. For random directed graphs, Hefetz, Steger and Sudakov (2014+) proved an analogue statement, but only for edge probability $p = \omega(\log n/\sqrt{n})$. In this paper we significantly improve their result to $p = \omega\left(\log^8 n/n\right)$, which is optimal up to the polylogarithmic factor.

This result is joint work with Asaf Ferber, Rajko Nenadov, Andreas Noever and Ueli Peter.