A tale of two properties of random graphs

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Abstract

Random graphs is an important family of combinatorial objects with several surprising properties. Some of them can be reproduced by deterministic structures (so-called quasirandom graphs). Many of these properties turn out to be equivalent and very recent results in combinatorics, such as the theory of graphons, shed new light on their mutual dependencies. In this talk we will focus on two properties which relation has not been fully understood. The first one is a global property stating that largest well-organized substructures of random graphs (cliques/stable sets in the undirected setting and transitive subtournaments in the directed setting) are of subpolynomial size. The second one is a local one saying that a sequence of random/quasirandom graphs cannot be defined by a forbidden pattern. The celebrated open Erdős-Hajnal Conjecture states that the first property implies the second one. I will show how probability theory, especially random graph theory, can be applied in the hunt for the counterexample to the Conjecture. All presented results are recent and involve several techniques in probabilistic graph theory such as Erdős-Renyi models (giving the best known upper bounds on the so-called Erdős-Hajnal coefficients of graphs), or quotient graphs with random base graphs (used to construct upper bounds on Erdős-Hajnal coefficients for graphs without nontrivial modules or with relatively small modules). Those probabilistic tools were crucial to prove that if true, the Conjecture is satisfied in a relatively weak sense. However they are still not refined enough to disprove the Conjecture. In the talk we will also present new positive results regarding the Conjecture, in particular the largest known infinite family of prime tournaments defined by a single ordering that satisfies the Conjecture. Finally we will show that the Erdős-Hajnal coefficients of all known prime Erdős-Hajnal tournaments H are of order at least $\Omega(\frac{1}{|H|^5 \log(|H|)})$. All previously known lower bounds were inversely proportional to the Szemeredi tower function. Not only does this new result enable us to significantly reduce the gap between best known lower and upper bounds for the coefficients but it leads to better understanding of the asymptotics of these important graph invariants.