Half-random Maker-Breaker games

We study Maker-Breaker positional games between two players, one of whom is playing randomly against an opponent with an optimal strategy. In both such scenarios, that is when Maker plays randomly and when Breaker plays randomly, we determine the sharp threshold bias of classical graph games, such as connectivity, Hamiltonicity, and minimum degree-k. The traditional, deterministic version of these games, with two optimal players playing, are known to obey the so-called probabilistic intuition. That is, the threshold bias of these games is asymptotically equal to the threshold bias of their random counterpart, where players just take edges uniformly at random. We find, that despite this remarkable agreement of the results of the deterministic and the random games, playing randomly against an optimal opponent is not a good idea: the threshold bias becomes significantly higher in favor of the "clever" player. An important qualitative aspect of the probabilistic intuition nevertheless carries through: for Maker to occupy a connected graph or a Hamilton cycle, the bottleneck is still the ability to achieve that there is no isolated vertex in his graph.

The talk represents joint work with Jonas Groschwitz