The phase transition in Achlioptas processes

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In the *Erdős–Rényi random graph process*, starting from an empty graph, in each step a new random edge is added to the evolving graph. One of its most interesting features is the 'percolation phase transition': as the ratio of the number of edges to vertices increases past a certain critical density, the global structure changes radically, from only small components to a single giant component plus small ones.

In this talk we consider *Achlioptas processes*, which have become a key example for random graph processes with dependencies between the edges. Starting from an empty graph these proceed as follows: in each step *two* potential edges are chosen uniformly at random, and using some rule *one* of them is selected and added to the evolving graph. We discuss why, for a large class of rules, the percolation phase transition is qualitatively comparable to the classical Erdős–Rényi process.

To be more concrete, for any so-called bounded-size rule $\mathcal{R}$ we, e.g., show that there exists a critical time t_c and constants $c, C > 0$ (depending on $\mathcal{R}$) such that the following holds. Assuming $\varepsilon^3 n \to \infty$ and $\varepsilon \to 0$ as $n \to \infty$, in the subcritical regime the size of the largest component whp satisfies

$$L_1(t_c n - \varepsilon n) \sim C\varepsilon^{-2} \log(\varepsilon^3 n),$$

and in the supercritical regime the largest component whp satisfies

$$L_1(t_c n + \varepsilon n) \sim c\varepsilon n.$$

In fact, we can establish these and related results *simultaneously* in every step outside of the critical window (which is, of course, much stronger than concentration in any specific step $m = t_c n \pm \varepsilon n$).

Based on joint work with Oliver Riordan.